RELATIONSHIPS BETWEEN CRITICAL STRESS INTENSITY FACTORS FOR UNIDIRECTIONAL COMPOSITES HAVING DIFFERENT REINFORCEMENT ANGLES

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(NASA-CR-158407) RELATIONSHIPS BETWEEN CRITICAL STRESS INTENSITY FACTORS FOR UNIDIRECTIONAL COMPOSITES HAVING DIFFERENT REINFORCEMENT ANGLES (Massachusetts Inst. of Tech.) 20 p

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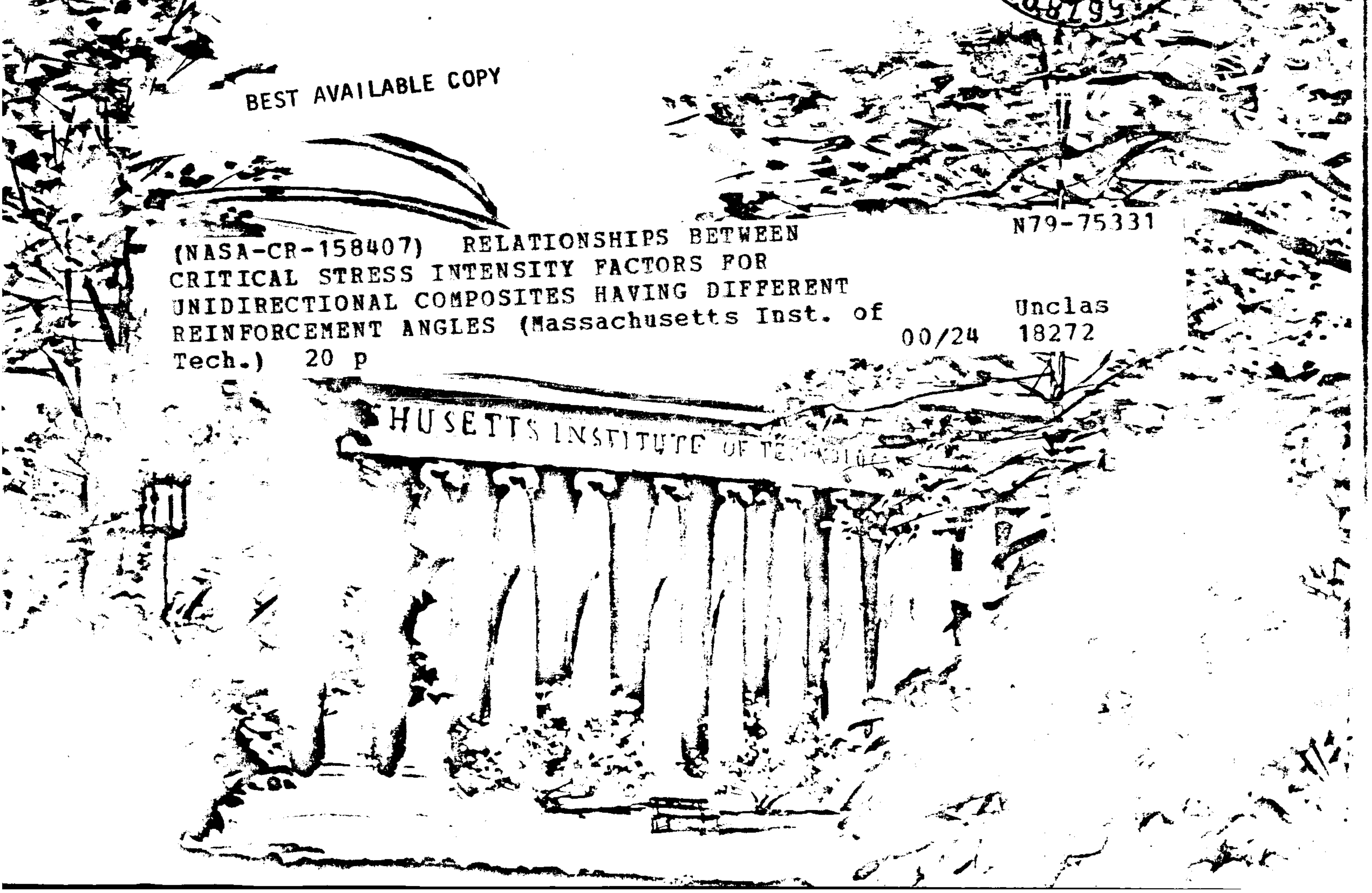

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Grant NSG-1349
Langley Research Center
National Aeronautics and Space Administration

ABSTRACT

A heterogeneous anisotropic model for notched fiber composites which was developed earlier by the authors is extended to derive relationships between the critical stress intensity factors of unidirectional fiber composites having different reinforcement angles. Under certain specified restrictions and given the critical stress intensity factor for one reinforcement angle, these relationships enable the calculation of the critical stress intensity factor for all reinforcement angles which exhibit the same mode of fracture.

INTRODUCTION

The purpose of this report is to describe the derivation of an expression relating the critical stress intensity factors of unidirectional composites having the same constitutents but different fiber reinforcement angles. The analysis is based on a crack-tip stress analysis model for unidirectional fiber composites by Kousiounelos and Williams† and assumes a brittle fracture criterion. For completeness of this report, the model assumptions and boundary conditions will be repeated here. Other fracture criteria can be used in the model to obtain a similar final expression which is valid only for composites which display the same general type of matrix fracture. We assume that brittle matrix fracture will occur when the maximum principal stress reaches a critical stress, regardless of the magnitude of the other principal stresses.

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†P.N. Kousiounelos and J.H. Williams, Jr., "Heterogeneous Anisotropic Model for Notched Fiber Composites", To appear in Fibre Science and Technology.

MODEL ASSUMPTIONS AND BOUNDARY CONDITIONS

The model for the specimen geometry, coordinates, and loading is shown in Fig. 1 and is based on the following set of assumptions:

1) The overall macroscopic behavior of the heterogeneous fiber composite is the same as the overall macroscopic behavior of the homogeneous orthotropic medium with appropriate equivalent elastic parameters.
2) The unidirectional fiber composite may be composed of thin parallel layers of fiber, matrix and coating material. See Fig. 2. (In this analysis, we shall assume without loss of utility that the coating material is not present.)
3) The extensional strain in the fiber direction near the crack tip is the same for the constituents of the heterogeneous strip model and the homogeneous orthotropic body at the same location.
4) The interfacial normal and shear stress components for the heterogeneous strip model (Fig. 3) are the same as the respective stress components predicted by the homogeneous orthotropic body at the same location. (It is important to note that these stresses are equivalent to the through-thickness-averaged stresses in the actual composite. A subsequent model which we are developing will

take into account through-thickness differences in the matrix and fiber stresses.)

5) The boundary conditions at the ends, $y = \pm \frac{L}{2}$ (Fig. 1) are uniform extensional strain ε_1 in the y-direction and zero shear strain in the xy plane. All other boundaries are stress free.

EQUIVALENT STRESS BOUNDARY CONDITIONS

It is convenient to express the specified strain boundary conditions in terms of equivalent stress boundary conditions. In the xy-coordinate system, the plane stress ($\sigma_z = 0$) constitutive relations are

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{21} & a_{22} & a_{26} \\ a_{61} & a_{62} & a_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \qquad (1)$$

where ε and γ are the normal and shear strains, respectively, and σ and τ are the normal and shear stresses, respectively. The a_{ij} constants are given in the Appendix.

In order to satisfy the specified strain boundary conditions on a general orthotropic body, as shown in Fig. 4, a combination of normal stress (σ_1') and shear stress (τ_{12}') must be applied along the edges $y = \pm \frac{L}{2}$. These stresses follow directly from the specified strain boundary conditions and equation (1) as

$$\sigma_1' = \frac{1}{E_1\left(a_{22} - \frac{a_{26}^2}{a_{66}}\right)} \quad \sigma_1 \tag{2}$$

and

$$\tau_{12}' = -\frac{a_{26}}{a_{66}} \frac{1}{E_1\left(a_{22} - \frac{a_{26}^2}{a_{66}}\right)} \quad \sigma_1 \, . \tag{3}$$

The normal stress σ_1 is a reference stress and is the stress required to produce the imposed extensional strain ε_1 in a unidirectional 0° composite. Thus, σ_1 is simply $(E_1 \; \varepsilon_1)$ where E_1 is the extensional elastic modulus. (See Appendix.)

CRACK-TIP STRESS AND STRAIN FIELDS

The crack-tip stress field in a homogeneous anisotropic body subjected to the boundary stresses σ_1' and τ_{12}' as shown in Fig. 4 is

$$\sigma_x = k\left(\frac{a}{b}\right)\left[\frac{\sigma_1'\sqrt{\pi a}}{(2\pi r)^{1/2}}\,\mathrm{Re}\left\{\frac{\mu_1\mu_2}{\mu_1-\mu_2}\left(\frac{\mu_2}{z_2^{1/2}}-\frac{\mu_1}{z_1^{1/2}}\right)\right\}\right.$$
$$\left.+\frac{\tau_{12}'\sqrt{\pi a}}{(2\pi r)^{1/2}}\,\mathrm{Re}\left\{\frac{1}{\mu_1-\mu_2}\left(\frac{\mu_2^2}{z_2^{1/2}}-\frac{\mu_1^2}{z_1^{1/2}}\right)\right\}\right]$$

$$\sigma_y = k\left(\frac{a}{b}\right)\left[\frac{\sigma_1'\sqrt{\pi a}}{(2\pi r)^{1/2}}\,\mathrm{Re}\left\{\frac{1}{\mu_1-\mu_2}\left(\frac{\mu_1}{z_2^{1/2}}-\frac{\mu_2}{z_1^{1/2}}\right)\right\}\right. \tag{4}$$
$$\left.+\frac{\tau_{12}'\sqrt{\pi a}}{(2\pi r)^{1/2}}\,\mathrm{Re}\left\{\frac{1}{\mu_1-\mu_2}\left(\frac{1}{z_2^{1/2}}-\frac{1}{z_1^{1/2}}\right)\right\}\right]$$

$$\tau_{xy} = k\left(\frac{a}{b}\right)\left[\frac{\sigma_1'\sqrt{\pi a}}{(2\pi r)^{1/2}}\,\mathrm{Re}\left\{\frac{\mu_1\mu_2}{\mu_1-\mu_2}\left(\frac{1}{z_1^{1/2}}-\frac{1}{z_2^{1/2}}\right)\right\}\right.$$
$$\left.+\frac{\tau_{12}'\sqrt{\pi a}}{(2\pi r)^{1/2}}\,\mathrm{Re}\left\{\frac{1}{\mu_1-\mu_2}\left(\frac{\mu_1}{z_1^{1/2}}-\frac{\mu_2}{z_2^{1/2}}\right)\right\}\right]$$

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† P.C. Paris and G.C. Sih, "Stress Analysis of Cracks", ASTM STP 381 1965.

where Re denotes the real part of a complex function, r and ϕ are the polar coordinates, and $k\left(\frac{a}{b}\right)$ is a specimen correction factor depending on the ratio of the crack length a to the specimen width 2b. (Refer to Fig. 1.) The μ_i are the roots of the equation

$$a_{11}\,\mu^4 - 2\,a_{16}\,\mu^3 + \left(2\,a_{12} + a_{66}\right)\mu^2 - 2\,a_{26}\,\mu + a_{22} = 0$$

and the z_i are defined by

$$z_i = (\cos\phi + \mu_i \sin\phi)^{\frac{1}{2}} .$$

The stress field of the homogeneous orthotropic composite with respect to the axes of elastic symmetry $(x'y')$ can be computed from equations (4) by a simple stress transformation. The corresponding strain field is

$$\varepsilon_{x'} = \frac{1}{E_2}\,\sigma_{x'} - \frac{\nu_{12}}{E_1}\,\sigma_{y'}$$

$$\varepsilon_{y'} = \frac{1}{E_1}\,\sigma_{y'} - \frac{\nu_{12}}{E_1}\,\sigma_{x'} \qquad (5)$$

$$\gamma_{x'y'} = \frac{1}{G_{12}}\,\tau_{x'y'} .$$

Using assumptions 3 and 4 and the material properties, the stresses in the composite constituents, along the fiber direction, are obtained as

$$\sigma_{fy'} = E_f \varepsilon_{y'} + \nu_f \sigma_{x'}$$

$$\sigma_{my'} = E_m \varepsilon_{y'} + \nu_m \sigma_{x'} , \tag{6}$$

where $\sigma_{fy'}$ and $\sigma_{my'}$ are the normal stresses in the fiber and matrix along the fiber direction, respectively; E_f and E_m are the fiber and matrix elastic moduli, respectively; and ν_f and ν_m are the Poisson's ratios for the fiber and matrix, respectively.

RELATION OF CRITICAL STRESS INTENSITY FACTORS FOR DIFFERENT REINFORCEMENT ANGLES

Equations (4) may be rewritten as

$$\sigma_x = \frac{K_I}{r^{1/2}} \cdot c \cdot F_x$$

$$\sigma_y = \frac{K_I}{r^{1/2}} \cdot c \cdot F_y \tag{7}$$

$$\tau_{xy} = \frac{K_I}{r^{1/2}} \cdot c \cdot F_{xy}$$

where

$$K_I \equiv \sigma_1' \sqrt{\pi a} \quad ; \qquad c \equiv \frac{k\left(\frac{a}{b}\right)}{\sqrt{2\pi}}$$

$$F_x = R_e\left[\frac{\mu_1\mu_2}{\mu_1-\mu_2}\left(\frac{\mu_2}{Z_2^{\frac{1}{2}}} - \frac{\mu_1}{Z_1^{\frac{1}{2}}}\right) - \frac{a_{26}}{a_{66}} \cdot \frac{1}{\mu_1-\mu_2}\left(\frac{\mu_2^{\,2}}{Z_2^{\frac{1}{2}}} - \frac{\mu_1^{\,2}}{Z_1^{\frac{1}{2}}}\right)\right]$$

$$F_y = R_e\left[\frac{1}{\mu_1-\mu_2}\left(\frac{\mu_1}{Z_2^{\frac{1}{2}}} - \frac{\mu_2}{Z_1^{\frac{1}{2}}}\right) - \frac{a_{26}}{a_{66}} \frac{1}{\mu_1-\mu_2} \cdot\left(\frac{1}{Z_2^{\frac{1}{2}}} - \frac{1}{Z_1^{\frac{1}{2}}}\right)\right]$$

$$F_{xy} = R_e\left[\frac{\mu_1\mu_2}{\mu_1-\mu_2}\left(\frac{1}{Z_1^{\frac{1}{2}}} - \frac{1}{Z_2^{\frac{1}{2}}}\right) - \frac{a_{26}}{a_{66}} \cdot \frac{1}{\mu_1-\mu_2} \cdot\left(\frac{\mu_1}{Z_1^{\frac{1}{2}}} - \frac{\mu_2}{Z_2^{\frac{1}{2}}}\right)\right]$$

The stress field in eqns (7) may be transformed to the x'y' axes, where the relative orientation of xy and x'y' is indicated in Fig. 4, as

$$\sigma_{x'} = \frac{K_I}{r^{\frac{1}{2}}} \cdot c \cdot G_{x'}$$

$$\sigma_{y'} = \frac{K_I}{r^{\frac{1}{2}}} \cdot c \cdot G_{y'} \qquad (8)$$

$$\tau_{x'y'} = \frac{K_I}{r^{\frac{1}{2}}} \cdot c \cdot G_{x'y'}$$

where

$$G_{x'} = \frac{1}{2}\left[\left(F_x+F_y\right) + \left(F_x-F_y\right)\cos(2\theta) - 2F_{xy}\sin(2\theta)\right]$$

$$G_{y'} = \frac{1}{2}\left[\left(F_x+F_y\right) - \left(F_x-F_y\right)\cos(2\theta) + 2F_{xy}\sin(2\theta)\right]$$

$$G_{x'y'} = \frac{1}{2}\left[\left(F_x-F_y\right)\sin(2\theta) + 2F_{xy}\cos(2\theta)\right].$$

The second of eqns.(5) may be used to eliminate $\varepsilon_{y'}$ from eqns.(6). Then, by substituting eqns.(8) into the resulting expression, the stresses in the fiber and the matrix along the fiber direction become

$$\begin{aligned} \sigma_{fy'} &= \frac{K_I}{r^{\frac{1}{2}}} \cdot c \cdot H_f \\ \sigma_{my'} &= \frac{K_I}{r^{\frac{1}{2}}} \cdot c \cdot H_m \end{aligned} \tag{9}$$

where

$$H_f = \left(\nu_f - \frac{E_f}{E_1}\nu_{12}\right) G_{x'} + \frac{E_f}{E_1} G_{y'}$$

$$H_m = \left(\nu_m - \frac{E_m}{E_1}\nu_{12}\right) G_{x'} + \frac{E_m}{E_1} G_{y'} \ .$$

In accordance with assumption 4 of the model formulation, the principal stress in the matrix is given by

$$\sigma_{m\ prin} = \frac{\sigma_{my'} + \sigma_{x'}}{2} + \left[\left(\frac{\sigma_{my'} - \sigma_{x'}}{2}\right)^2 + \tau^2_{x'y'}\right]^{\frac{1}{2}}. \tag{10}$$

Substitution of eqns.(8) and (9) into eqn.(10) gives

$$\sigma_{m\ prin} = \frac{K_I}{r^{\frac{1}{2}}} \cdot c \cdot R_m \tag{11}$$

where

$$R_m = \frac{H_m + G_{x'}}{2} + \left[\left(\frac{H_m - G_{x'}}{2}\right)^2 + G^2_{x'y'}\right]^{\frac{1}{2}}.$$

similarly the principal stress in the fiber is

$$\sigma_{f\ prin} \doteq \frac{K_I}{r^{1/2}} \cdot c \cdot R_f \tag{12}$$

where

$$R_f = \frac{H_f + G_{x'}}{2} + \left[\left(\frac{H_f - G_{x'}}{2}\right)^2 + G^2_{x'y'}\right]^{1/2} .$$

If for each fiber orientation of the set of unidirectional composites under consideration there is a common fracture mode, namely brittle matrix fracture, then in accordance with the assumed fracture criterion, fracture will initiate in each composite of the same material at the same critical value of $\sigma_{m\ prin}$. This critical value of $\sigma_{m\ prin}$ is assumed to occur within a damage-zone radius r_{Ym} which is analogous to a plastic-zone radius in metals. At this point, r_{Ym} will not be discussed except to remark that r_{Ym} must be much less than a in accordance with the foregoing use of LEFM. The equivalence of the critical principal matrix stresses may be written as

$$\left[\left(\sigma_{m\ prin}\right)_{\theta i}\right]_{cr} = \left[\left(\sigma_{m\ prin}\right)_{\theta j}\right]_{cr} \tag{13}$$

where θi and θj represent angles for which the above restrictions hold. By eqn.(11), eqn.(13) may be expressed as

$$\left[\left(\frac{K_I}{r_{Ym}^{1/2}} \cdot c \cdot R_m\right)_{\theta i}\right]_{cr} = \left[\left(\frac{K_I}{r_{Ym}^{1/2}} \cdot c \cdot R_m\right)_{\theta j}\right]_{cr} . \tag{14}$$

If i and j also represent different specimens and if they possess the same geometric configuration, eqn. (14) reduces to

$$\left[\left(\frac{K_I}{r_{Ym}^{1/2}} \cdot R_m\right)_{\theta i}\right]_{cr} = \left[\left(\frac{K_I}{r_{Ym}^{1/2}} \cdot R_m\right)_{\theta j}\right]_{cr} . \tag{15}$$

Furthermore, if $\left(r_{Ym}\right)_{\theta i} = \left(r_{Ym}\right)_{\theta j}$, eqn. (15) becomes

$$\left(\frac{K_{I_{\theta i}}}{K_{I_{\theta j}}}\right)_{cr} = \left(\frac{R_{m_{\theta j}}}{R_{m_{\theta i}}}\right). \tag{16}$$

$K_{I_{\theta i}}$ may be considered as an experimentally determined critical stress intensity factor. Then, $K_{I_{\theta j}}$ represents a theoretically determinable critical stress intensity factor for any other angle of the composite for which the same mode of fracture prevails. The $R_{m_{\theta k}}$ are functions of the composite consituents and the stress analysis, and may be computed indepently of the fracture properties of the composite, in accordance with the reference on page 2 of this report.

APPENDIX

EXPRESSIONS FOR THE a_{ij} CONSTITUTIVE COEFFICIENTS

The coefficients a_{ij} for an orthotropic material at an angle θ (Fig. 4) with respect to its axes of elastic symmetry are,

$$a_{11} = \frac{\cos^4\theta}{E_2} + \left[\frac{1}{G_{12}} - 2\,\frac{\nu_{12}}{E_1}\right] \sin^2\theta\, \cos^2\theta + \frac{\sin^4\theta}{E_1}$$

$$a_{22} = \frac{\sin^4\theta}{E_2} + \left[\frac{1}{G_{12}} - 2\,\frac{\nu_{12}}{E_1}\right] \sin^2\theta\, \cos^2\theta + \frac{\cos^4\theta}{E_1}$$

$$a_{12} = \left[\frac{1}{E_1} + \frac{1}{E_2} + 2\,\frac{\nu_{12}}{E_1} - \frac{1}{G_{12}}\right] \sin^2\theta\, \cos^2\theta - \frac{\nu_{12}}{E_1}$$

$$a_{16} = \left[2\left(\frac{\sin^2\theta}{E_1} - \frac{\cos^2\theta}{E_2}\right) + \left(\frac{1}{G_{12}} - 2\,\frac{\nu_{12}}{E_1}\right)\left(\cos^2\theta - \sin^2\theta\right)\right] \sin\theta\, \cos\theta$$

$$a_{26} = \left[2\left(\frac{\cos^2\theta}{E_1} - \frac{\sin^2\theta}{E_2}\right) - \left(\frac{1}{G_{12}} - 2\,\frac{\nu_{12}}{E_1}\right)\left(\cos^2\theta - \sin^2\theta\right)\right] \sin\theta\, \cos\theta$$

$$a_{66} = 4\left[\frac{1}{E_1} + \frac{1}{E_2} + 2\frac{\nu_{12}}{E_1} - \frac{1}{G_{12}}\right]\sin^2\theta\cos^2\theta + \frac{1}{G_{12}}$$

$$a_{21} = a_{12}$$

$$a_{61} = a_{16}$$

$$a_{62} = a_{26}$$

E_1, E_2 are the composite elastic moduli along and perpendicular to the fiber direction, respectively, ν_{12} is the composite Poisson's ratio for loading along the fiber direction, and G_{12} is the in-plane composite shear modulus.

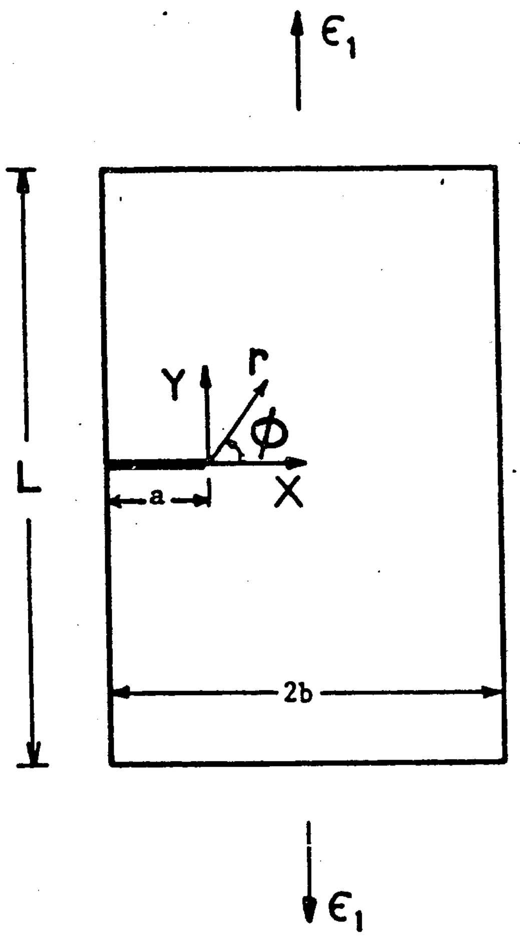

Fig. 1 Single-edge notched specimen showing geometry, loading and crack-tip coordinates.

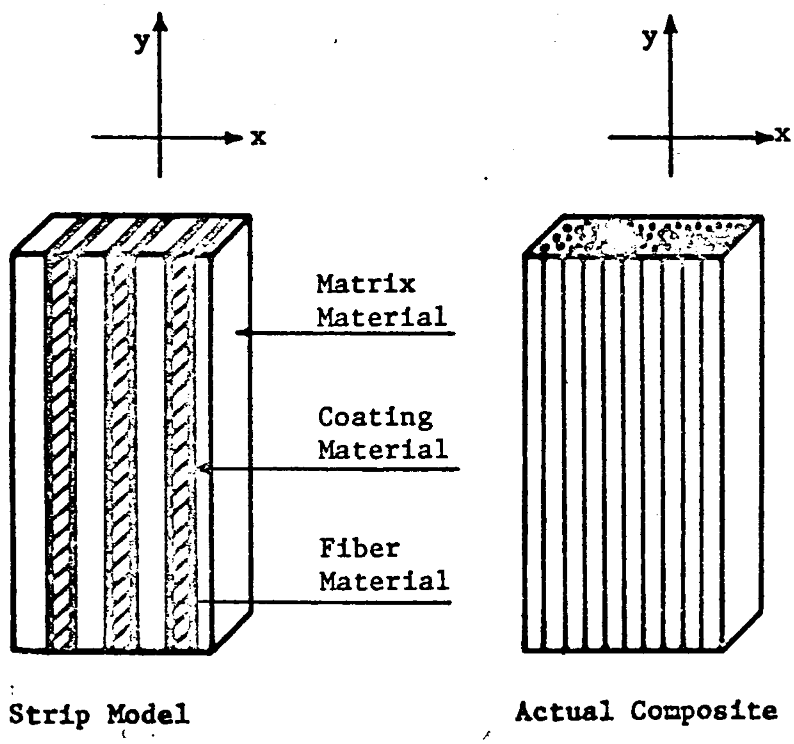

Fig. 2 The heterogeneous composite strip model as compared with the actual composite.

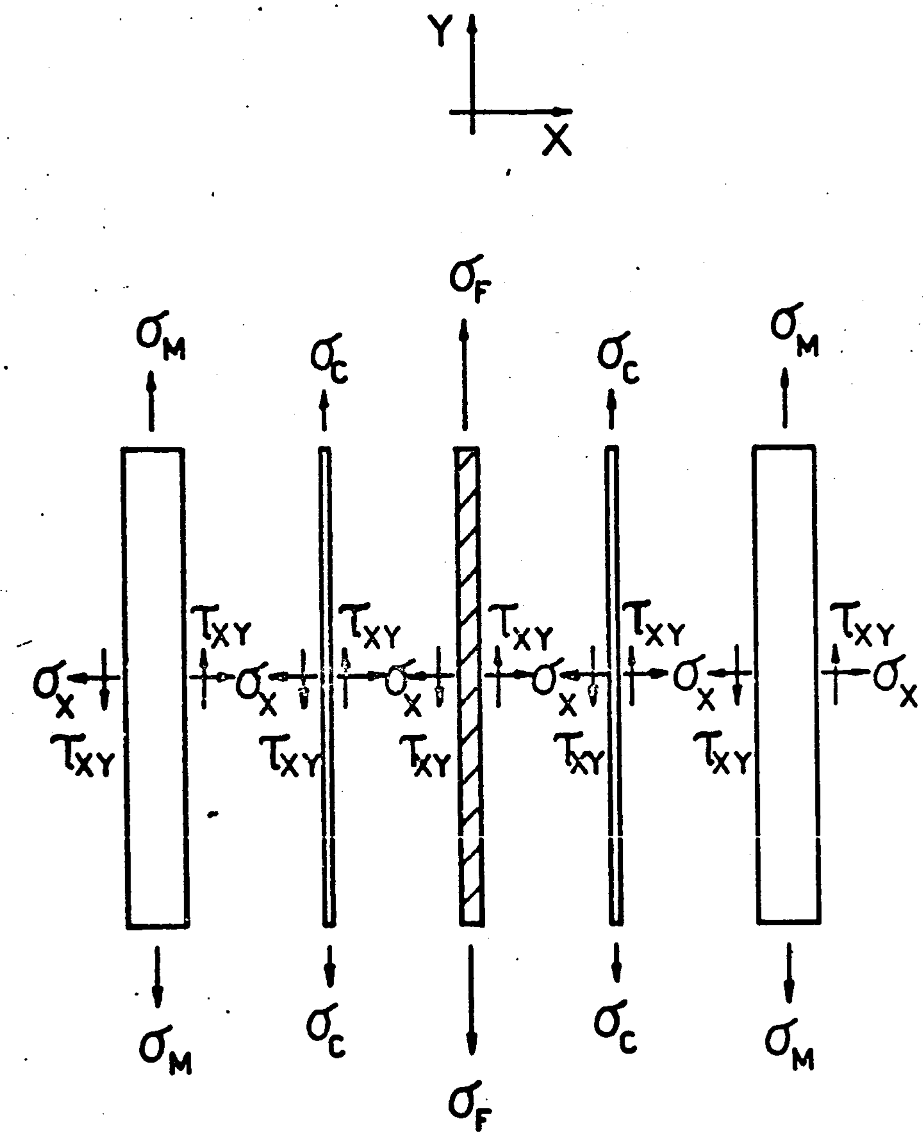

Fig. 3 The stresses in the multilayered composite model. (The subscripts M, C and F refer to the matrix, coating and fiber, respectively.)

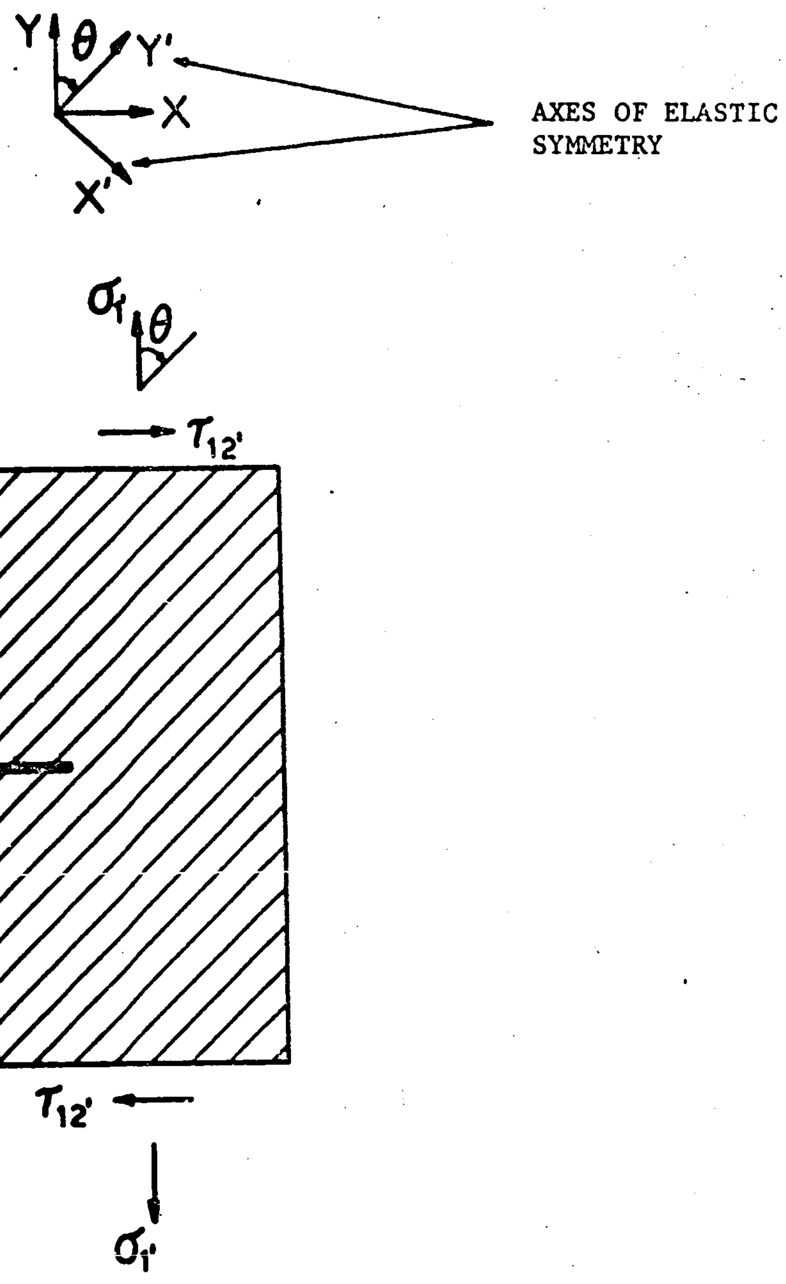

Fig. 4 General orthotropic body with x'y'-axes of elastic symmetry at the angle θ with respect to the system xy-axes.